

Stability beyond the neutron drip-line near the third peak of the r-process nucleosynthesis

M.M. SHARMA AND A.A. SALDANHA

Physics Department, Kuwait University, Kuwait 13060

Abstract

We have investigated the nuclear shell effects at $N = 126$ in the region of the third peak of the r -process nucleosynthesis within the framework of the relativistic mean-field theory using the Lagrangian model NL-SV1 with the vector self-coupling of ω -meson. Our study encompasses even-even nuclei with $N = 110 - 140$ in the isotopic chains of Hf ($Z = 72$) down to Ba ($Z = 56$). It is shown that the nuclear shell effects at $N = 126$ remain strong even as one moves far away from the line of the β -stability. As the neutron drip line approaches $N = 126$, nuclei exhibit vanishingly small neutron separation energy. However, going beyond the neutron drip line, we observe an interesting feature in that some nuclei near $N \sim 132 - 134$ for the isotopic chains of $Z = 62 - 68$ show enhanced neutron separation energy. This is especially pronounced for the isotopes of Gd ($Z = 64$) and Dy ($Z = 66$). These nuclei exhibit the phenomenon of stability beyond the neutron drip line. Our analysis of the single-particle spectrum shows that this is engendered by the deformation assumed by these nuclei with the consequence that the neutron single-particle spectrum is pushed down in energy, thus leading to enhanced stability beyond the drip line.

I. INTRODUCTION

About half of nuclei heavier than Fe are synthesized in the process of rapid neutron capture (r-process) (Burbidge *et al.* 1957, Hillebrandt 1978, Kratz *et al.* 1993). Extremely neutron-rich nuclei with ~ 10 -30 neutrons away from the stability line are produced in environments of high neutron densities and high temperatures. Due to extremely large isospin, these nuclei are highly unstable and are experimentally inaccessible, especially those in the heavy mass region. Undergoing a sequence of neutron capture accompanied by β -decays, these extremely neutron-rich nuclei give birth to heavy elements in the nature. The r-process path passes through the magic numbers $N = 50, 82$ and 126 at different mass values. The synthesis of nuclei around these magic numbers is reflected vividly in the known nuclear abundance peaks around $A \sim 80, 130$ and 190 , respectively.

The shell effects at the magic numbers play a crucial role in determining the r-process nuclear abundances (Kratz *et al.* 1993). The question whether the shell effects near the r-process path are strong or do quench has become crucial to understanding the nucleosynthesis of heavy nuclei (Pfeiffer *et al.* 2001). It was argued for long that a quenching of the shell effects at $N = 82$ near the r-process path was required in order to reproduce r-process abundances in the second peak (Pfeiffer 1997). This argument came primarily from the ability of the macroscopic-microscopic mass formula Extended Thomas-Fermi with Strutinsky Integral (Strutinsky 1968) built with quenching based upon Bogoliubov pairing (ETF-SI(Q)) (Pearson *et al.* 1996) to reproduce r-process abundances. This quenching has been termed as artificial, for it is based upon the extreme quenching due to the Skyrme force SkP (Dobaczewski *et al.* 1984). In contrast, microscopic theories such as the RMF theory with the Hartree-Bogoliubov approach have not shown any significant weakening of the shell effects near the r-process path (Sharma and Farhan 2002).

Recently, indications have emerged from analysis of several experimental data that shell quenching which was invoked earlier is not required. Anomalous behaviour of 2^+ excitation energies in neutron-rich Cd isotopes was cited as an evidence of shell quenching of $N = 82$ (Dillman *et al.* 2002). However, the recent work by Rodriguez *et al.* (2008) has shown that there is no need to assume quenching of $N = 82$ shell closure to explain the experimental finding. This is further corroborated by the recent experimental work (Jungclaus *et al.* 2007, Gorska *et al.* 2009), where results on level scheme of ^{130}Cd show no evidence of quenching at

$Z = 48$. These works have confirmed the prediction of the absence of quenching at $N = 82$ shell closure for $Z = 48$ and neighbouring elements by one of the authors and a collaborator (Sharma and Farhan 2002) with the force NL-SV1 within the RMF theory. On the other hand, the shell effects at $N = 82$ were shown to weaken only near the neutron drip line. In that work (Sharma and Farhan 2002), it was shown that force NL-SV1 based upon the vector self-coupling of ω -meson (Sharma *et al.* 2000), which reproduces the shell effects in nuclei at the stability line is able to reproduce the available data on the waiting-point nucleus ^{80}Zn (Sharma and Farhan 2002).

The shell effects at $N=126$ near the r-process path were also investigated (Farhan and Sharma 2006) within the framework of the Relativistic Hartree-Bogoliubov theory with the Lagrangian model NL-SV1 using spherical configuration of nuclei. In contrast to the $N=82$ shell effects, it was shown that the shell strength at $N=126$ remains strong even in approaching the neutron drip line. The spherical configuration of nuclei is appropriate for nuclei in the vicinity of a shell closure especially when it remains robust. However, away from the shell closure nuclei assume deformation.

In the present work, we have investigated the ground-state properties of nuclei across the shell closure $N = 126$ within the framework of the deformed RMF theory. In Section 2, we present a brief description of the formalism of the RMF theory. Section 3 provides the detailed results of the study of the rare-earth nuclei spanning the shell closure $N = 126$. The final section provides the summary and conclusions.

II. THE RELATIVISTIC MEAN-FIELD THEORY: FORMALISM

The RMF approach (Serot and Walecka 1986) is based upon the Lagrangian density which consists of fields due to the various mesons interacting with the nucleons. The mesons include the isoscalar scalar σ -meson, the isovector vector ω -meson and the isovector vector ρ -meson. The Lagrangian density is given by:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \\ & \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}g_4(\omega_\mu\omega^\mu)^2 - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\not{\omega}\psi - g_\rho\bar{\psi}\not{\rho}\boldsymbol{\tau}\psi - e\bar{\psi}\mathbf{A}\psi \end{aligned} \quad (1)$$

The bold-faced letters indicate the vector quantities. Here M , m_σ , m_ω and m_ρ denote the nucleon-, the σ -, the ω - and the ρ -meson masses respectively, while g_σ , g_ω , g_ρ and $e^2/4\pi = 1/137$ are the corresponding coupling constants for the mesons and the photon, respectively.

The σ meson is assumed to move in a scalar potential of the form:

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4. \quad (2)$$

This was introduced by Boguta and Bodmer (1977) in order to make a substantial improvement in the surface properties of finite nuclei. This Ansatz for the σ -potential has since become a standard and necessary ingredient for description of the properties of finite nuclei.

In this work, we have employed the non-linear vector self-coupling of the ω -meson, in addition to the non-linear scalar potential of Eq. (2). The coupling constant for the non-linear ω -term is denoted by g_4 in the Lagrangian (1).

The field tensors of the vector mesons and of the electromagnetic field take the following form:

$$\begin{aligned} \Omega^{\mu\nu} &= \partial^\mu\omega^\nu - \partial^\nu\omega^\mu \\ \mathbf{R}^{\mu\nu} &= \partial^\mu\boldsymbol{\rho}^\nu - \partial^\nu\boldsymbol{\rho}^\mu \\ F^{\mu\nu} &= \partial^\mu\mathbf{A}^\nu - \partial^\nu\mathbf{A}^\mu \end{aligned} \quad (3)$$

The mean-field approximation constitutes the lowest order of the quantum field theory. Herein, the nucleons are assumed to move independently in the meson fields. The latter are replaced by their classical expectation values. The ground-state of the nucleus is described by a Slater determinant $|\Phi\rangle$ of single-particle spinors ψ_i ($i = 1, 2, \dots, A$). The stationary state solutions ψ_i are obtained from the coupled system of Dirac and Klein-Gordon equations. The variational principle leads to the Dirac equation:

$$\{-i\alpha\nabla + V(\mathbf{r}) + \beta[m*]\} \psi_i = \epsilon_i\psi_i \quad (4)$$

where $V(\mathbf{r})$ represents the *vector* potential:

$$V(\mathbf{r}) = g_\omega\omega_0(\mathbf{r}) + g_\rho\tau_3\rho_0(\mathbf{r}) + e\frac{1-\tau_3}{2}A_0(\mathbf{r}) \quad (5)$$

and $S(\mathbf{r})$ is the *scalar* potential

$$S(\mathbf{r}) = g_\sigma\sigma(\mathbf{r}) \quad (6)$$

which defines the effective mass as:

$$m^*(\mathbf{r}) = m + S(\mathbf{r}) \quad (7)$$

The Klein-Gordon equations for the meson fields are time-independent inhomogeneous equations with the nucleon densities as sources.

$$\begin{aligned} \{-\Delta + m_\sigma^2\}\sigma(\mathbf{r}) &= -g_\sigma\rho_s(\mathbf{r}) - g_2\sigma^2(\mathbf{r}) - g_3\sigma^3(\mathbf{r}) \\ \{-\Delta + m_\omega^2\}\omega_0(\mathbf{r}) &= g_\omega\rho_v(\mathbf{r}) + g_4\omega^3(\mathbf{r}) \\ \{-\Delta + m_\rho^2\}\rho_0(\mathbf{r}) &= g_\rho\rho_3(\mathbf{r}) \\ -\Delta A_0(\mathbf{r}) &= e\rho_c(\mathbf{r}) \end{aligned} \quad (8)$$

For the case of an even-even nucleus with time-reversal symmetry, the spatial components of the vector fields, $\boldsymbol{\omega}$, $\boldsymbol{\rho}_3$ and A vanish. For the mean-field, the nucleon spinors provide the corresponding source terms:

$$\begin{aligned} \rho_s &= \sum_{i=1}^A \bar{\psi}_i \psi_i \\ \rho_v &= \sum_{i=1}^A \psi_i^+ \psi_i \\ \rho_3 &= \sum_{p=1}^Z \psi_p^+ \psi_p - \sum_{n=1}^N \psi_n^+ \psi_n \\ \rho_c &= \sum_{p=1}^Z \psi_p^+ \psi_p \end{aligned} \quad (9)$$

where the sums are taken over the valence nucleons only. Consequently, the ground-state of a nucleus is obtained by solving the coupled system of the Dirac and Klein-Gordon equations self-consistently.

A. Axially deformed RMF

We solve the Dirac equation as well as the Klein-Gordon equations by expansion of the wavefunctions into a complete set of eigen solutions of an harmonic oscillator potential (Gambhir et al. 1990) In the axially symmetric case the spinors f_i^\pm and g_i^\pm are expanded in terms of the eigenfunctions of a deformed axially symmetric oscillator potential

$$V_{osc}(z, r_\perp) = \frac{1}{2}M\omega_z^2 z^2 + \frac{1}{2}M\omega_\perp^2 r_\perp^2. \quad (10)$$

Taking volume conservation into account, the two oscillator frequencies $\hbar\omega_\perp$ and $\hbar\omega_z$ can be expressed in terms of a deformation parameter β_0 .

$$\hbar\omega_z = \hbar\omega_0 \exp\left(-\sqrt{\frac{5}{(4\pi)}}\beta_0\right) \quad (11)$$

$$\hbar\omega_{\perp} = \hbar\omega_0 \exp\left(+\frac{1}{2}\sqrt{\frac{5}{(4\pi)}}\beta_0\right). \quad (12)$$

The corresponding oscillator length parameters are given by

$$b_z = \sqrt{\frac{\hbar}{M\omega_z}} \quad \text{and} \quad b_{\perp} = \sqrt{\frac{\hbar}{M\omega_{\perp}}}. \quad (13)$$

The volume conservation gives $b_{\perp}^2 b_z = b_0^3$. The basis is now determined by the two constants $\hbar\omega_0$ and β_0 , which are chosen optimally.

The deformation parameter of the oscillator basis β_0 is chosen to be identical for the Dirac spinors and the meson fields. The deformation parameter β_2 is obtained from the calculated quadrupole moments for protons and neutrons through

$$Q = Q_n + Q_p = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} A R_0^2 \beta_2 \quad (14)$$

with $R_0 = 1.2A^{1/3}$ (fm). The quadrupole moments is calculated as

$$Q_{n,p} = \langle 2r^2 P_2(\cos\theta) \rangle_{n,p} = 2\langle 2z^2 - x^2 - y^2 \rangle_{n,p}. \quad (15)$$

III. DETAILS OF CALCULATIONS

In this work we have employed the Lagrangian model NL-SV1 (Sharma et al. 2000) with the inclusion of the vector self-coupling of ω -meson in the RMF Lagrangian. It was shown (Sharma et al. 2000) that inclusion of the vector self-coupling of ω -meson provides an improved description of the shell effects in nuclei along the stability line. The force NL-SV1 was developed with a view to improve the predictions of the ground-state properties of nuclei, such as binding energies, charge radii and isotopes shifts of nuclei along the stability line and far away from it. The parameters of the set NL-SV1 are given in Table I.

The input parameters required to carry out explicit numerical calculations are: neutron pairing gap Δ_n , the proton pairing gap Δ_p and the number of oscillator shells N_F and N_B of the fermionic wavefunctions and meson fields, respectively. Both the fermionic and bosonic wavefunctions have been expanded in a basis of 20 harmonic oscillator shells in this work.

For pairing gaps, we have used the formula due to Möller and Nix (1992) as given by:

$$\begin{aligned} \Delta_n &= 4.8 N^{-1/3} \\ \Delta_p &= 4.8 Z^{-1/3}. \end{aligned} \quad (16)$$

TABLE I: The Lagrangian parameters of the force NL-SV1 (Sharma et al. 2000) used in the RMF calculations.

Parameters	NL-SV1
M	939.0
m_σ	510.0349
m_ω	783.0
m_ρ	763.0
g_σ	10.1248
g_ω	12.7266
g_ρ	4.4920
g_2	-9.2406
g_3	-15.388
g_4	41.0102

An axially symmetric deformed configuration with reflection symmetry has been assumed for nuclei. For each nucleus, RMF minimization has been sought both in the prolate as well as in the oblate region of the deformation space.

IV. RESULTS AND DISCUSSION

As mentioned in the introduction, the nuclei corresponding to the third peak in the r-process nucleosynthesis lie in the region of the rare-earths in the periodic table. Especially, it concerns the rare-earth nuclei beyond the magic number $N = 126$ with elements from $Z = 56$ to $Z = 72$. We have then investigated nuclei in this region using the RMF theory with the Lagrangian model NL-SV1.

A. The binding energies

The binding energy per nucleon of nuclides in various isotopic chains studied in this work is shown for the lowest energy minimum (ground state) in Fig. 1. Fig. 1(a) shows the

binding energy per nucleon for the chains of Hf ($Z = 72$), Yb ($Z = 70$), Er ($Z = 68$) and Dy ($Z = 66$), and Fig. 1(b) shows those for the chains Gd ($Z = 64$), Sm ($Z = 62$), Nd ($Z = 60$) and Ce ($Z = 58$). The curves show a natural decline in the binding energy as the neutron number increases, for one is already treading farther and farther away from the line of β -stability as the neutron number increases. The difference in the relative binding energies for the various chains is also evident as the curves show a decrease as the isospin increases from one chain to the other (with a decrease in Z) for a given neutron number.

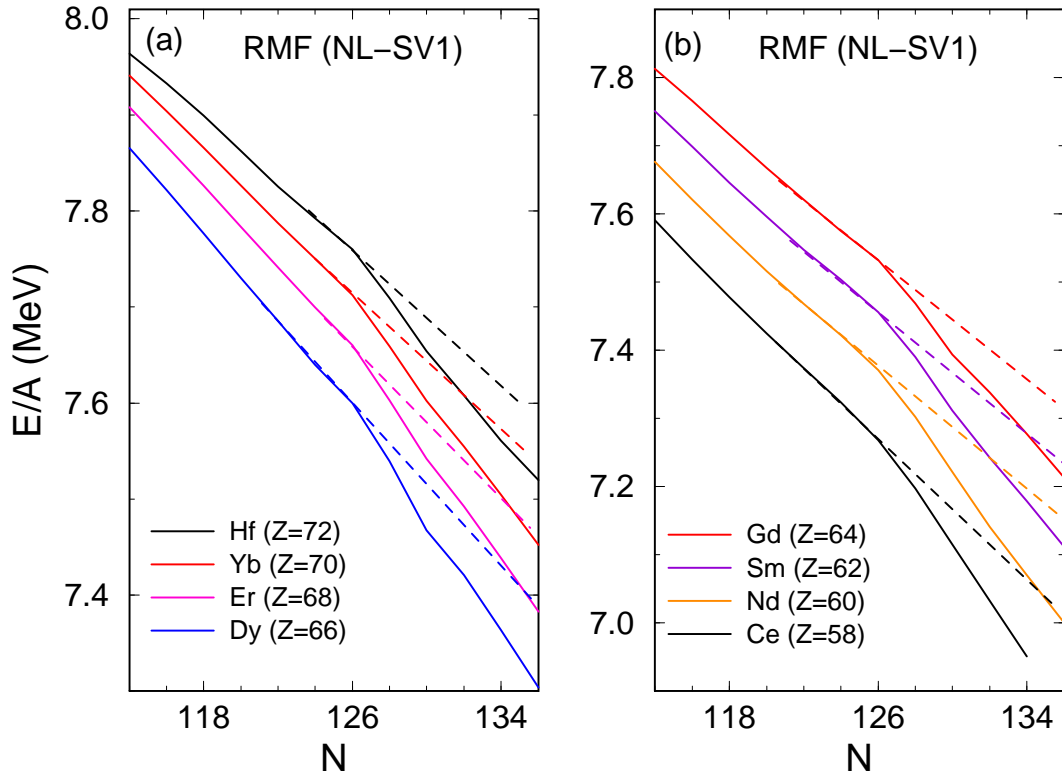


FIG. 1: The binding energies of nuclei obtained with the RMF theory using the Lagrangian set NL-SV1 with the vector self-coupling of ω -meson. The kink at $N = 126$ is seen clearly. The view is augmented by the dashed line at each curve, which has been drawn just to guide the eye.

The behaviour of the binding energies at the magic number $N = 126$ is conspicuous. The effect of the shell-closure is evident in the kink at $N = 126$. A downward and a larger slope above $N = 126$ shows a rapid decrease in the binding energy for nuclides as compared to those below $N = 126$. This change in the slope is demonstrated by the dashed line which has been drawn to guide the eye. The divergence from the straight line (dashed) increases

slightly especially for the isotopic chains below $Z = 66$. This is due to the fact (as we will see in the later sections below) that nuclides with $N > 126$ approach the drip line as one moves to the chains below $Z = 66$. This indicates that for nuclides which are closer to the drip line, there is a rapid decline in the binding energy contribution due to the added neutrons.

B. The deformation properties

The nuclides in the isotopic chains studied in this work fall in the region of the rare-earths as mentioned earlier. It is well known that nuclei in this region assume rather strong deformations. Hence this work is intended to investigate the properties of nuclei in this region in the deformed RMF theory. As a result of axially deformed RMF minimizations, we have obtained deformation properties of nuclei. We show the ensuing quadrupole deformation β_2 of nuclides in various isotopic chains in Figs. 2 and 3. The figures are arranged in the descending order of atomic number Z with a view to visualizing the effect of moving from the region closer to the stability line to that approaching the neutron drip line.

Fig. 2 shows the β_2 values for the isotopic chains of Hf ($Z = 72$), Yb ($Z = 70$), Er ($Z = 68$) and Dy ($Z = 66$). Nuclides in these chains especially below the magic number $N = 126$ are still away from the neutron drip line. The neutron drip line in these chains is expected to lie far beyond the neutron number $N = 140$.

Nuclides in all the isotopic chains in Fig. 2 exhibit an identical pattern of prolate shape followed by a transition to an oblate shape, which is followed by spherical shape near and at $N = 126$, which is followed by a prolate shape again for nuclides above $N = 126$. Nuclides in these chains show a significant prolate deformation below $N = 120$. For the Hf chain, several nuclides exhibit a prolate deformation which continues from $N = 110$ until $N = 122$. This is striking that Hf nuclides show more extended region of the prolate deformation in contrast to its neighbours with $Z = 70$, $Z = 68$ and $Z = 66$. For the Hf chain, even the nuclide with $N = 122$ which is so close to the magic number $N = 126$ exhibits a prolate deformation. This is followed by a shape transition to a weakly deformed oblate shape at $N = 124$ followed by sphericity for the magic nucleus $N = 126$.

For the isotopic chains $Z = 70$ and especially for $Z = 68$ and $Z = 66$, a fewer nuclei exhibit prolate shape as compared with $Z = 72$. For $Z = 70$, the prolate-to-oblate transition takes place at $N = 120$, whereas this is shifted further down to $N = 116$ for $Z = 68$ and

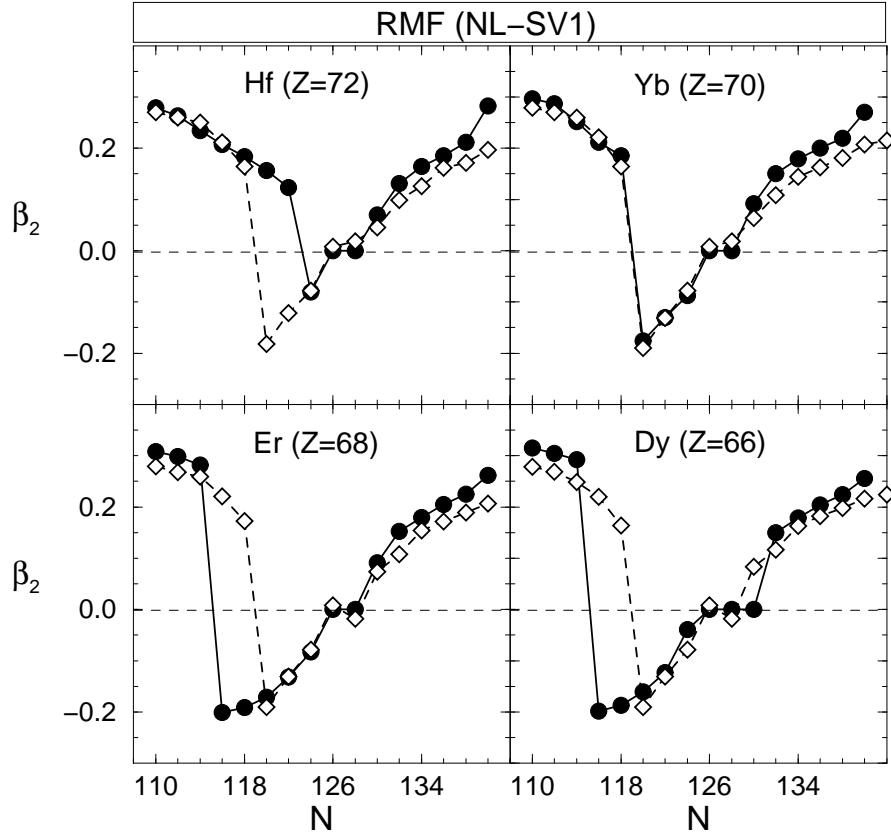


FIG. 2: The quadrupole deformation β_2 for the lowest minimum (the ground state) obtained in the RMF theory with NL-SV1 (full circles) for the isotopic chains $Z = 66 - 72$. A comparison is made with the FRDM values (open diamonds).

$Z = 66$. Consequently, the region of oblate shape increases to 3 nuclei for $Z = 70$ and to 5 nuclei for $Z = 68$ and $Z = 66$ before achieving sphericity at $N=126$. However, for the chains with $Z = 68 - 72$ in Fig. 2(a) only two nuclides, i.e., with $N = 126$ and $N = 128$ exhibit sphericity. For the Dy ($Z = 66$) chain, it increases to three nuclides which are spherical near $N = 126$. This would imply that the shell effect is in no way becoming milder in going down from Hf ($Z = 72$) to Dy ($Z = 66$), i.e. the shell gap at $N = 126$ retains its strong character or rather it might even imply a somewhat strengthening of the shell effect which induces more sphericity in the vicinity of $N = 126$.

For all the four chains in Fig. 2, nuclei above $N = 128$ exhibit a prolate shape with increasing magnitude of deformation as one goes further away from the magic number $N =$

126. This is natural, for the effect of the closed shell gets diminished as one moves farther away from the shell closure.

A comparison of the β_2 values from the macroscopic-microscopic mass formula FRDM (Möller et al. 1994) is made in Fig. 2. The FRDM shows a remarkably similar feature such as transition from prolate to oblate shape as the neutron number increases followed by a spherical shape near and at $N = 126$, which is then followed by a prolate shape as one moves beyond $N = 126$. Incidentally, the number of nuclei exhibiting spherical shape near $N = 126$ with FRDM is nearly the same as with NL-SV1. This may be due to the fact that the shell effects at $N = 126$ with FRDM are similar to those with NL-SV1. This was already shown to be the case in the spherical relativistic Hartree-Bogoliubov approach (Farhan and Sharma 2006) with NL-SV1, where a comparison was made with FRDM.

The difference between the predictions of NL-SV1 and those of FRDM are primarily about the point of transition from the prolate to the oblate shape below $N = 126$. For all the isotopic chains in Fig. 2, the prolate-oblate transition takes place at $N = 120$. Thus, there is a near constancy about it. In comparison, the transition point is shifting to lower values of N as one moves from $Z = 72$ to $Z = 66$ with NL-SV1. This can be attributed to changing structural factors as one traverses from one value of Z to another.

The quadrupole deformation β_2 for the chains Gd ($Z = 64$), Sm ($Z = 62$), Nd ($Z = 60$) and Ce ($Z = 58$) is shown in Fig. 3. The prolate-to-oblate transition is exhibited by all these chains as in Fig. 2. The transition point, however, is shifted a few neutron number, i.e., it takes place at $N = 114$ for all the chains with $Z = 58 - 64$. Consequently, only a fewer nuclides exhibit a prolate shape below $N = 126$. As a result, more nuclei exhibit oblate shape below $N = 126$. At the same time, the number of nuclei exhibiting a spherical shape near the magic number increases from 4 ($N = 124 - 130$) for $Z = 64$ and to 6 ($N = 122 - 132$) for $Z = 62$. Incidentally, for these isotopic chains, magic nuclei and those beyond it are approaching the drip line closely as we will see below in the subsection on 2-neutron separation energies. However, in spite of the proximity of nuclei to the magic number $N = 126$ for the chains $Z=62-66$ non-magic nuclei near $N=126$ are not susceptible to a deformation. Also, the number of spherical nuclei near $N = 126$ is higher than those in Fig. 2. This is due to the reason that in going to the lower Z values, the strong shell character of the magic number $N = 126$ is maintained even as one approaches the neutron drip line.

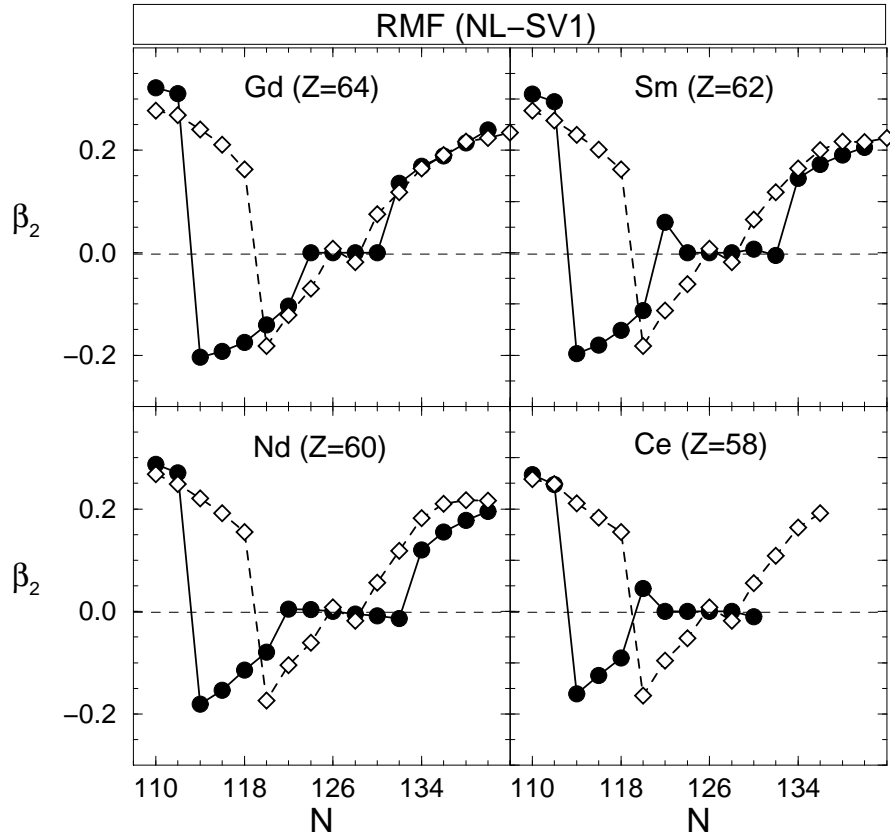


FIG. 3: The quadrupole deformation β_2 for the lowest minimum (the ground state) obtained in the RMF theory with NL-SV1 (full circles) for the isotopic chains $Z = 58 - 64$. A comparison is made with the FRDM values (open diamonds).

A comparison of the β_2 values due to NL-SV1 with those from FRDM shows that the basic feature of the shape transitions in Fig. 3 is similar to that in Fig. 2. The FRDM values show the prolate-oblate transition point at $N = 120$ as in Fig. 2, whereas it has shifted to $N = 114$ for NL-SV1 for all the chains in Fig. 3.

C. The shape-coexistence near $N = 126$

The phenomena of shape-coexistence is well known in nuclides in several parts of the periodic table. It arises due to interplay of deformation and the consequent re-adjustment of single-particle levels. A large number of nuclides in the isotopic chains we have investigated

also exhibit the phenomenon of the shape-coexistence of prolate-oblate shapes. We show the nuclides exhibiting the shape-coexistence in this region in Figs. 4 and 5.

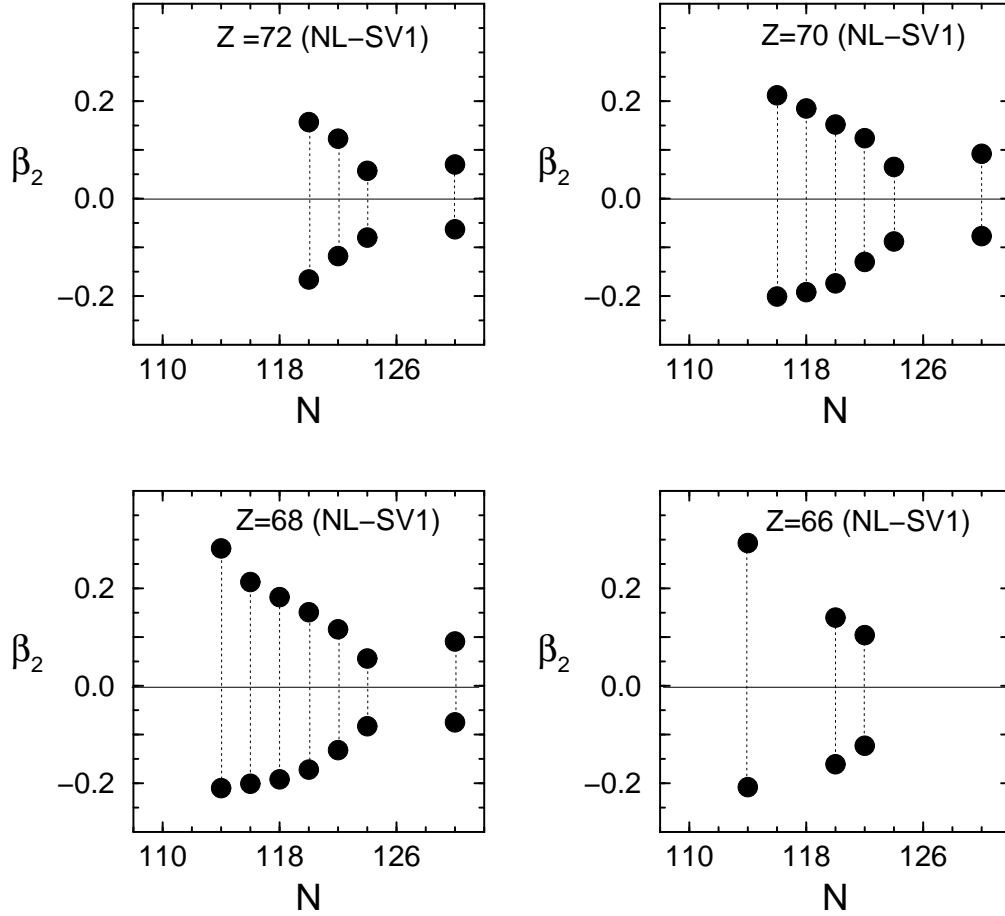


FIG. 4: The shape-coexistence of prolate and oblate shapes in the ground state of nuclei in the isotopic chains with $Z = 66 - 72$.

The shape-coexistence for nuclides in the isotopic chains of $Z = 66 - 72$ are shown in Fig. 4. Most of the occurrence of the shape-coexistence in these chains takes place in nuclides below the magic number $N = 126$. It is striking that a significantly large number of nuclides exhibit the shape-coexistence for the chains $Z = 70$ (Yb) and $Z = 68$ (Er) and especially those in the latter. For the Yb chain, the isotopes with $N = 116 - 124$ exhibit the shape-coexistence, whereas for the Er chain it takes place for the isotopes with $N = 114 - 124$.

The magnitude of deformation for prolate and the corresponding coexisting oblate shape is comparable and modest. A reduction in the value is shown as one approaches the shell-closure. It is interesting to note that nuclides with $N = 124$, which are so close to the shell-

closure, are amenable to a shape-coexistence albeit with a significantly reduced deformation values. For the isotopic chains $Z = 72$ (Hf) and $Z = 66$ (Dy), in comparison, there are only a limited cases of shape-coexistence below $N = 126$. In contrast, there is only one case each for the shape-coexistence in the chains $Z = 72$, $Z = 70$ and $Z = 68$ for nuclides with neutron number above $N = 126$. For $Z = 66$, there is no case of shape-coexistence above $N=126$.

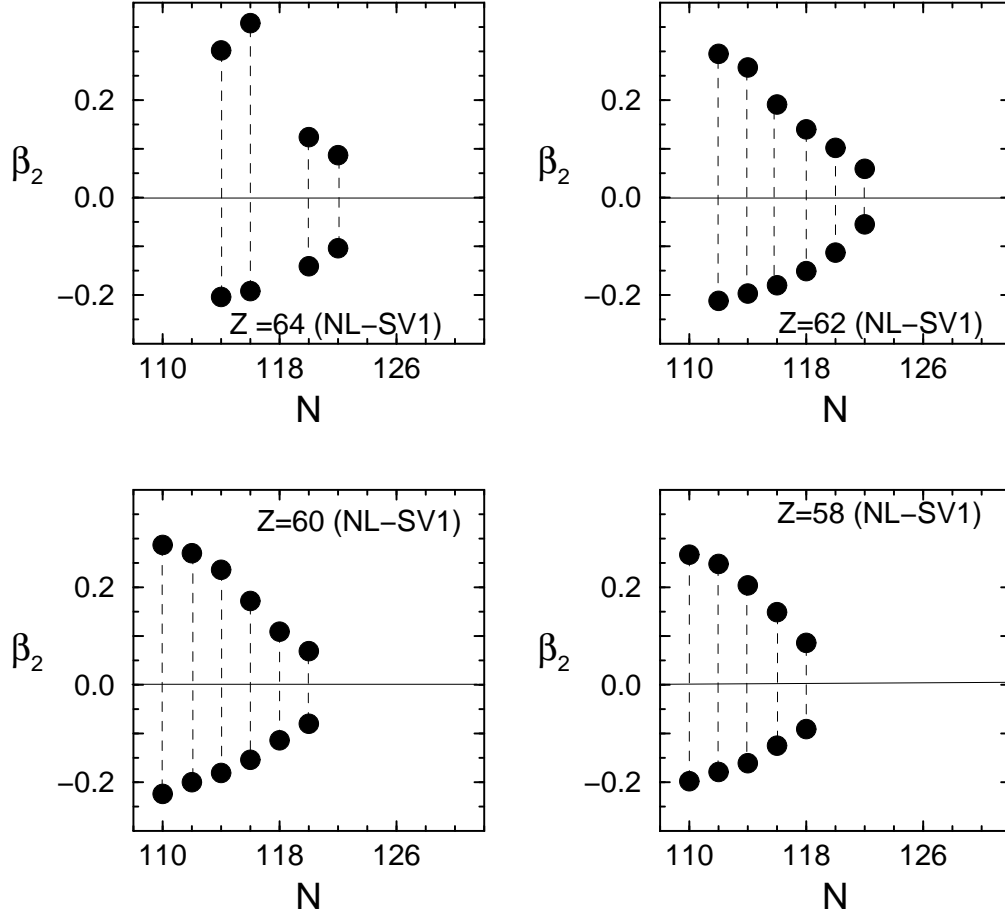


FIG. 5: The shape-coexistence of prolate and oblate shapes in the ground state of nuclei in the isotopic chains with $Z = 58 - 64$.

The shape-coexistence in the isotopic chains $Z = 64$ (Gd), $Z = 62$ (Sm), $Z = 60$ (Nd) and $Z = 58$ (Ce) is shown in Fig. 5. For these chains, there is no shape-coexistence found for neutron numbers above $N = 126$. Just as in the case of $Z = 66$ (Dy) in Fig. 4, its neighbouring chain $Z = 64$ (Gd) exhibits a fewer cases of shape-coexistence as compared to the chains $Z = 62$ (Sm) and $Z = 60$ (Nd) and to some extent $Z = 58$ (Ce). The picture

for the shape-coexistence in Fig. 5 is very similar to that in Fig. 4. Nearly all even-even isotopes in the chains $Z = 62$ exhibit shape-coexistence from $N = 112 - 122$. This is shifted to $N = 110 - 120$ for the $Z = 60$ chain and to $N = 110 - 118$ for the $Z = 58$ chain.

There are no cases of shape-coexistence seen for $N > 126$ for any of the chains in Fig. 5. As noted above, nuclides with $N > 126$ are gradually moving towards the neutron drip-line as Z is decreasing from $Z = 64$ to $Z = 58$. This is evident also from Fig. 2 on deformations whereby nuclides with $N \geq 126$ are projected to be spherical.

In summarizing the shape-coexistence in Fig. 4 and 5, it is fair to say that the phenomenon of shape-coexistence is abound in a large number of nuclides in the region $N = 110 - 124$ below the magic number. This implies that the potential energy landscape for nuclides in this region of the periodic table is relatively 'softer' in the deformation space.

D. The two-neutron separation energies

The shell effects near $N = 126$ being the main objective of this work, we show 2-neutron separation energies S_{2n} in Fig. 6. The S_{2n} values show a plateau like curves for all isotopic chains below the magic number $N = 126$, with slight undulations here and there due to modulations caused by the deformation. Yet, most of the curves show a slight declining trend in going from $N = 112$ to $N = 126$. Also, the level of each curve is decreasing as the atomic number Z of an isotopic chain decreases.

The S_{2n} values demonstrate a sudden decline at $N = 128$ for all the isotopic chains. This is a manifestation of the shell closure at $N = 126$. The decrease in the slope of the curve between $N = 126$ and $N = 128$ amongst various isotopic chains is minimal, implying that the shell gap at $N = 126$ does not show a significant change in going from $Z = 70$ to $Z = 56$. This means that the shell gap at $N = 126$ remains rather intact even as one approaches the nuclei near the drip line. There is thus no washing out (quenching) of the shell effects at $N = 126$ near the neutron drip line. A similar result was obtained within the framework of the spherical relativistic Hartree-Bogoliubov approach in our previous work (Farhan and Sharma, 2006).

For nuclides with $N = 130$ for the isotopic chains $Z = 66$ to $Z = 56$, there is a steep decline in the S_{2n} values approaching towards a vanishing value. This indicates that for $Z < 66$ there is an imminent arrival of the neutron drip line as one goes above $N = 126$.

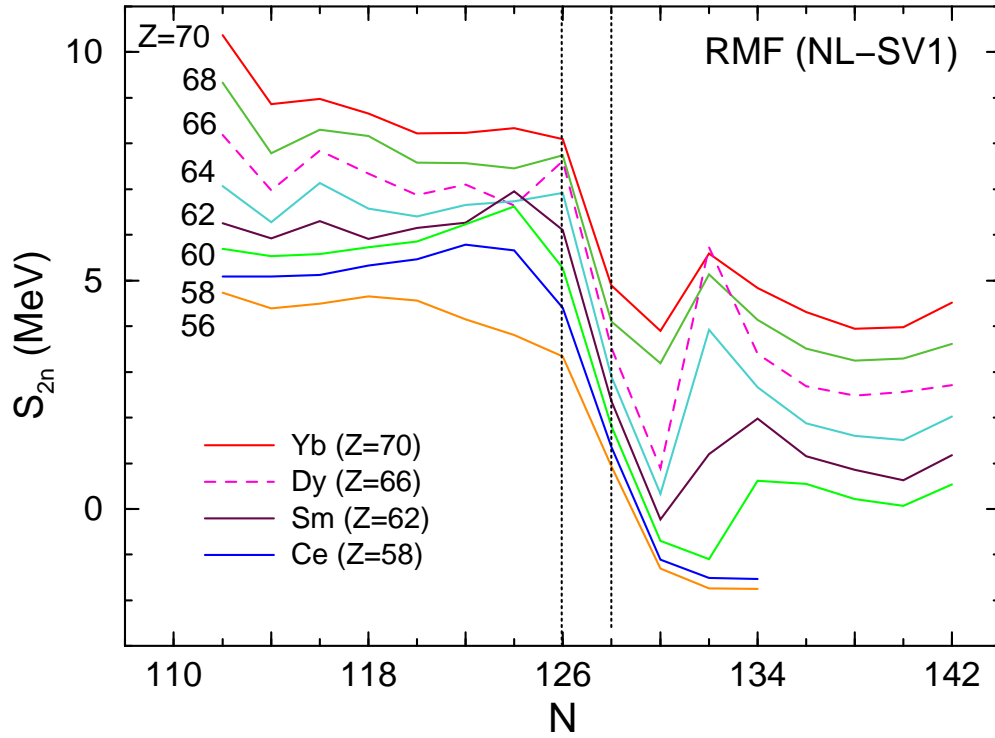


FIG. 6: The 2-neutron separation energies of nuclei in the isotopic chains $Z = 56 - 70$ obtained with the deformed RMF calculations using the Lagrangian model NL-SV1.

E. Stability beyond the neutron drip-line

In going above $N = 130$, we observe a spectacular increase in the S_{2n} values. This is especially the case for the isotopic chains Dy ($Z = 66$), Gd ($Z = 64$) with a peak at $N = 132$ and a bit lesser for the chains of Sm ($Z = 62$) and Nd ($Z = 60$) showing a peak at $N = 134$. To some extent, this effect is also visible for Yb ($Z = 70$) and Er ($Z = 68$) with a peak at $N = 132$. The sudden spurt in the S_{2n} value especially for Dy and Sm isotopes with $N = 132$ signals a significantly higher binding of additional neutrons as compared to $N = 130$ isotopes. This provides an additional binding energy to an isotope as compared to its lighter neighbour. Thus, this phenomenon can suitably be termed as '*stability beyond the neutron drip line*'. In fact, due to the advent of this feature, the neutron drip line for some of these isotope chains is extended farther in the neutron space. To our understanding, this is the first observation of the stability beyond the neutron drip line. In the subsection below, we will see as to how this phenomenon is produced.

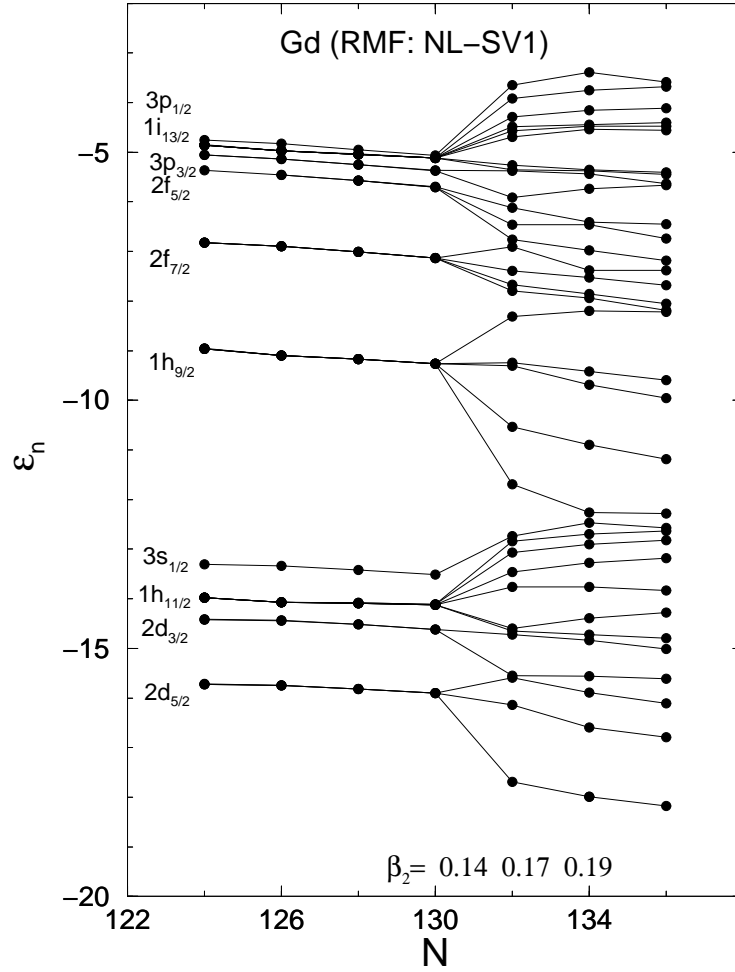


FIG. 7: The Nilsson single-particle levels for the isotopes of Gd ($Z = 64$) obtained with the Lagrangian model NL-SV1 in the RMF theory. The isotopes above $N = 128$ are deformed as indicated by the β_2 values shown at the bottom of the panel.

F. The neutron single-particle levels

In order to visualize as to how the stability beyond the neutron drip line is produced, we show the neutron single-particle levels (Nilsson levels) for the isotopes of the Gd ($Z = 64$) chain in Fig. 7. The isotopes with $N=124-130$ are all spherical and hence there is no splitting of the j -levels. However, in going above $N = 130$, the nuclei assume a modest deformation with $\beta_2 \sim 0.14 - 0.19$. The effect of this deformation is clearly evident in the splitting of j -levels into various Ω -orbitals.

The salient feature of the deformed single-particle levels in Fig. 7 is that some of the

Ω -levels are pushed down in the energy, thus effectively lowering the 'centre-of-gravity' of the neutron j -levels for the isotopes with $N = 132 - 136$ as compared to the isotope with $N = 130$. This downward shift in the energy levels is then responsible for the additional binding energy provided by the deformation and thus bestowing an additional stability beyond the neutron drip line.

V. SUMMARY AND CONCLUSIONS

We have investigated the nuclear shell effects at the magic number $N = 126$ in the vicinity of the r -process path for the third peak in the r -process abundances. This region passes through the rare-earth region encompassing $Z = 56 - 72$. Within the framework of the relativistic mean-field theory, we have calculated the ground-state properties of even-even nuclei in these isotopic chains. The force NL-SV1 with the nonlinear vector self-coupling of ω -meson has been employed in this work.

The results show that a large number of nuclides attain deformation except those close to the magic number $N = 126$. The shape transitions from oblate-prolate-spherical from $N = 110$ to $N = 126$ are exhibited by nearly all the chains with the point of transitions shifting slightly. We observe that a large number of nuclei especially those below $N = 126$ exhibit the phenomenon of shape-coexistence of prolate and oblate shapes in the ground state.

Analyzing the 2-neutron separation energy of nuclei, it is seen that the shell gap at $N = 126$ does not show much softening in going from the region away from the drip line towards the neutron drip line. Thus, the shell effects at $N = 126$ remains rather strong even in the vicinity of the drip line. This is consistent with an earlier work (Farhan and Sharma, 2006) wherein nuclear shell effects were studied within the framework of the relativistic Hartree-Bogoliubov theory with spherical shapes. As nuclei near and at the magic number are all spherical as seen in the present work, the result on the shell effects in the present work in the deformed RMF theory is not expected to differ from the earlier work (Farhan and Sharma, 2006).

The most interesting aspect that has emerged from this work is the increase in the 2-neutron separation energy of nuclides near $N = 132 - 134$ in several isotopic chains and especially for Gd ($Z = 64$) and Dy ($Z = 66$) as compared to their lighter counterparts. It

is shown that these nuclei exhibit the phenomenon akin to a stability beyond the neutron drip line. The analysis of the deformed single-particle levels for these nuclides shows that due to the splitting of levels caused by the deformation, several Ω -levels are pushed down in energy, thus lowering down the single-particle levels effectively. This lends an additional binding energy to nuclei though these nuclei are very close to the neutron drip line.

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